REPORT No. 238

THE EFFECT OF FLIGHT PATH INCLINATION ON AIRPLANE VELOCITY

By WALTER S. DIEHL
Bureau of Aeronautics

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SUMMARY

This report was prepared at the request of the National Advisory Committee for Aeronautics in order to supply a systematic study of the relations between the flight velocity V and its horizontal component V_H, in power glides. Curves of V and V_H plotted against the inclination of the flight path θ are given, together with curves which show the maximum values of V_H and the corresponding values of θ. Curves are also given showing the effect of small departures from the horizontal in high speed performance testing.

INTRODUCTION

While it is obvious that the speed of an airplane must be greater along a downwardly inclined flight path than along a horizontal flight path, there has not been in the past any great necessity for a systematic study of the relations involved, but in view of the probable interest which such a systematic solution would now hold, for pilots in particular, it has been undertaken.

The extent of previous calculations along this line appear to have been limited to the determination of the diving speed, a problem not considered in this study since it has been very well treated elsewhere. For example, see British Advisory Committee for Aeronautics, R. & M. No. 492 (Vol. II, 1917–18).

Owing to the nature of the variables, particularly the thrust, no strictly exact solution can be obtained for the relation between the angle of the flight path and the flight speed. The results are sufficiently exact for all practical purposes, however. Such inaccuracy as might be introduced by the assumption as to variation of thrust with speed would not be large except when the velocity was very large or very small.

In this report the phrase "Horizontal flight" means flight at constant altitude, in accordance with common usage.

THE EFFECT OF INCLINATION OF THE FLIGHT PATH ON HORIZONTAL VELOCITY

The forces acting on an airplane which is flying with full power along a path inclined $\theta°$ to the horizontal are the thrust T, the drag D, the lift L, and the weight W. Let it be assumed that

(1) The drag varies as the square of the velocity, i. e., $D=KV^2$.

(2) The thrust varies linearly with velocity and acts along the flight path.

Resolving the forces along the flight path gives $T-W\sin\theta=D=KV^2$ (1)

θ being considered positive for an upward inclination.

Consider an average airplane in horizontal flight at the maximum speed V_m. If the flight speed be increased the engine will speed up and deliver slightly more power, but since the increase in N is less rapid than the increase in V there will be an increase in V/ND and a decrease in the propeller efficiency η. The relation between thrust and speed may be conveniently studied by means of the coefficient $C_4=\frac{2\pi Q}{\rho V^2 D^3}$. Figure 3 of National Advisory Committee for Aeronautics Technical Report No. 186 contains a plot which gives the values of C_4 for varying ratio p/D and V/ND, so that for any p/D a curve of C_4 vs. V/ND may be drawn. Q and V are the only variables in C_4, and since the variation of Q with N and with V may be estimated closely, the value of C_4, V/ND and N may be obtained for any assumed value of V.

A study of two propellers gives the following results:

$\frac{V}{V_m}$	1.00	1.20	1.40	1.60	1.80	2.00
$\frac{p}{D}=0.5$ $\frac{N}{N_0}$	1.00	1.06	1.14	1.22	1.30	1.39
$\frac{p}{D}=0.5$ $\frac{T}{T_0}$	1.00	.85	.71	.54	.35	.14
$\frac{p}{D}=1.0$ $\frac{N}{N_0}$	1.00	1.10	1.21	1.32	1.43	1.54
$\frac{p}{D}=1.0$ $\frac{T}{T_0}$	1.00	.89	.78	.66	.54	.42

The values of T/T_0 plotted against V/V_m lie on or near two straight lines, which allows T to be expressed in terms of V by a simple linear relation.

When $V=2V_m$, T may be between zero and $T_0/2$ depending upon the propeller and engine characteristics, the average value probably being nearer to zero than to $T_0/2$.

Consequently

$$T=T_0-a(V-V_m)$$

and

$$T=\frac{T_0}{2} \text{ when } V=2V_m$$

therefore

$$a=\frac{T_0}{V_m}$$

and

$$T=2T_0-\frac{T_0V}{V_m} \tag{2}$$

Substituting Equation (2) in (1) gives

$$V^2=\frac{2T_0}{K}-\frac{T_0V}{KV_m}-\frac{W}{K}\sin\theta$$

or, since

$$T_0=KV_m^2 \text{ and } \frac{W}{T_0}=\left(\frac{L}{D}\right)_0$$

$$V^2=2V_m^2-VV_m+\left(\frac{L}{D}\right)_0 V_m^2\sin\theta \tag{3}$$

from which

$$V=V_m\left[-\frac{1}{2}+\sqrt{2.25-\left(\frac{L}{D}\right)_0\sin\theta}\right] \tag{4}$$

V is the velocity along the flight path which is inclined at the angle θ to the horizontal. The horizontal component of V is

$$V_H=V_m\left[-\frac{1}{2}+\sqrt{2.25-\left(\frac{L}{D}\right)_0\sin\theta}\right]\cos\theta \tag{5}$$

If the assumption be made that $T=T_0/2$ when $V=2V_m$, Equations (4) and (5) become

$$V=V_m\left[-\frac{1}{4}+\sqrt{1.5625-\left(\frac{L}{D}\right)_0\sin\theta}\right] \tag{4a}$$

and

$$V_H=V_m\left[-\frac{1}{4}+\sqrt{1.5625-\left(\frac{L}{D}\right)_0\sin\theta}\right]\cos\theta \tag{5a}$$

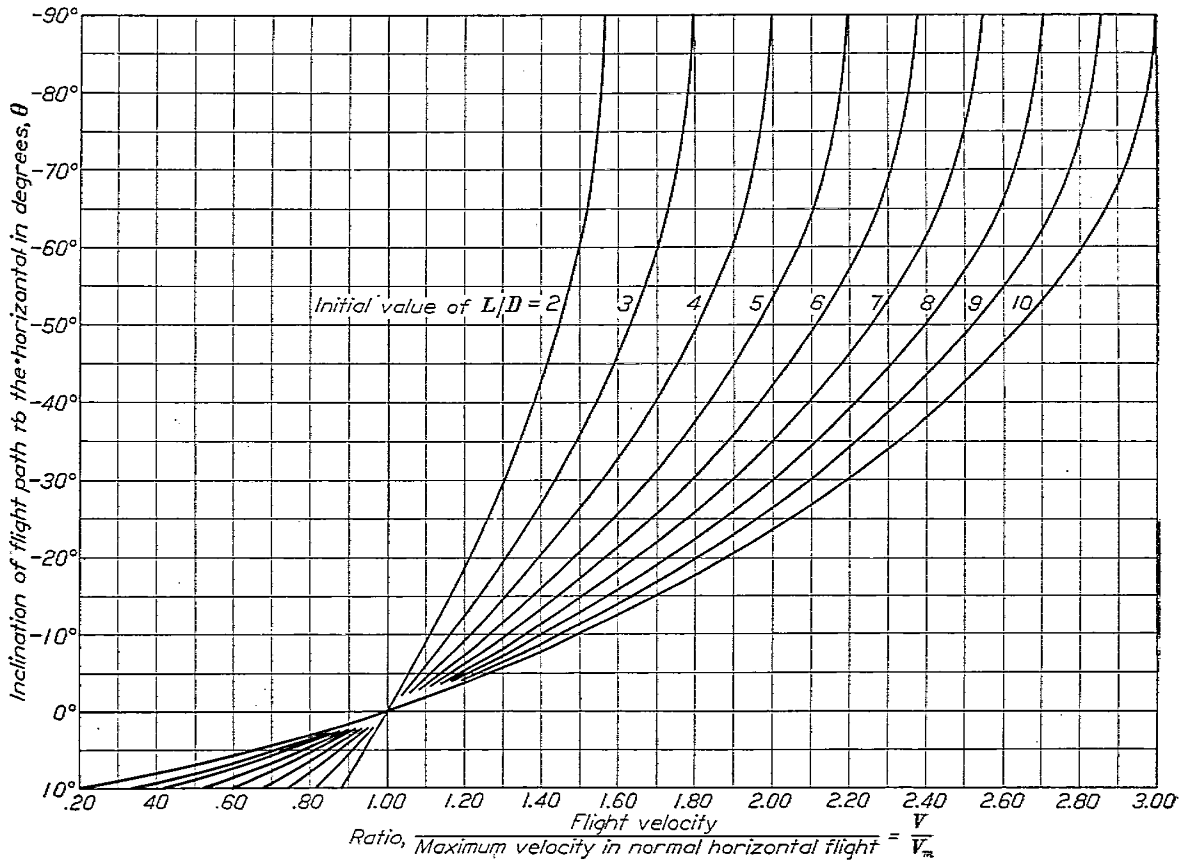

FIG. 1.—Effect of inclination of flight path on flight velocity.
Note: θ is negative when the flight path is inclined downward. (As in a glide)
$T=0$ when $V=2\,V_m$

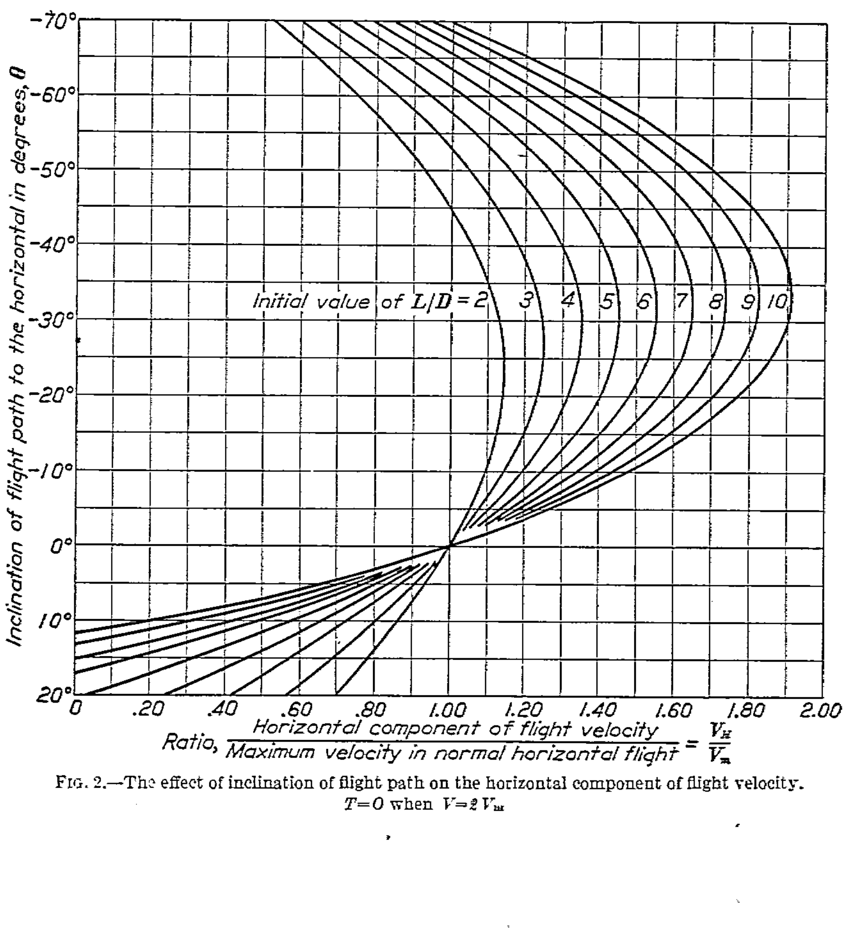

FIG. 2.—The effect of inclination of flight path on the horizontal component of flight velocity.
$T=0$ when $V=2\,V_m$

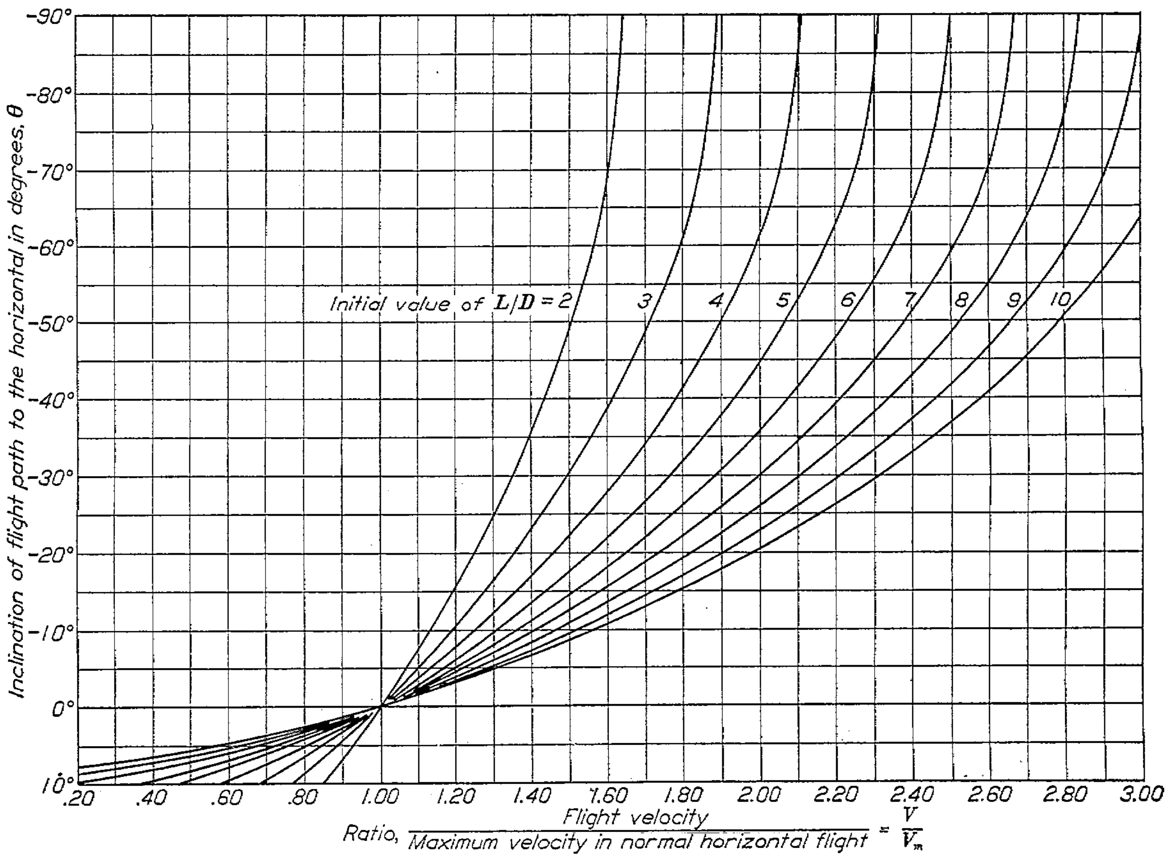

FIG. 3.—Effect of inclination of flight path on flight velocity.
Note: θ is negative when the flight path is inclined downward. (As in a glide)
$T=\frac{T_o}{2}$ when $V=2\,V_m$

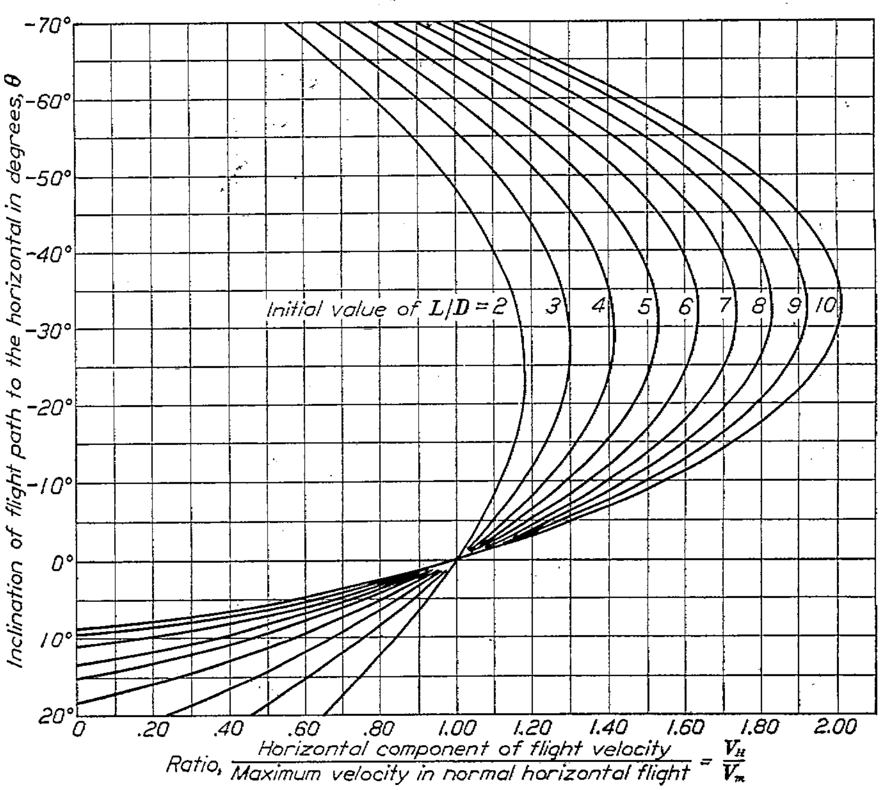

FIG. 4.—The effect of inclination of flight path on the horizontal component of flight velocity
$T=\frac{T_o}{2}$ when $V=2\,V_m$

INCREASE IN HORIZONTAL VELOCITY DUE TO DOWNWARD INCLINATION OF THE FLIGHT PATH

When an airplane flies with full power along a flight path which is inclined at $\theta°$ downward from the horizontal, the component of the weight ($L/D \sin \theta$) in Equation (5) is positive and increases the velocity along the flight path. An inspection of Equation (5) shows that the horizontal component of the flight velocity first increases, reaches a maximum value, and then decreases as θ is increased.

Tables I, II, III, and IV contain values of flight velocity and the ratio of the horizontal component of the flight velocity to the maximum velocity in sustained horizontal flight as calculated from Equations (4), (4a), (5), and (5a) for various values of $(L/D)_0$ and θ. These values are plotted on Figures 1, 2, 3, and 4.

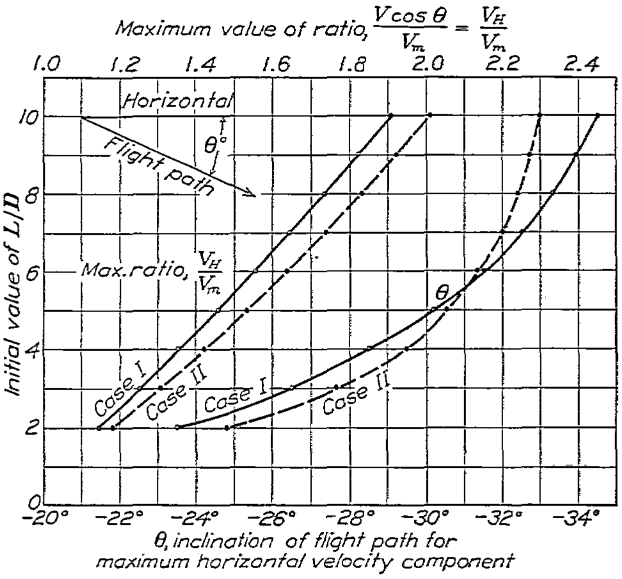

FIG. 5.

A study of Figures 2 and 4 discloses several interesting and instructive facts. For example, the maximum increase in effective horizontal velocity which may be obtained by flying along an inclined flight path is considerably greater than might be expected, since it is of the order of 40 per cent for the average airplane. It is also surprising to note that this maximum increase occurs for values of θ of the order of $-30°$. Equation (5) does not readily lend itself to the analytical calculation of the value of θ giving the maximum ratio V_H/V_m. These have therefore been obtained graphically from large-scale plots of Figures 2 and 4 and the values so obtained are given in Table V and plotted on Figure 5.

Figures 2 and 4 also show that for the average airplane θ must be of the order of $-60°$ before the horizontal component of the flight velocity becomes less than the maximum velocity which can be sustained in horizontal flight.

EFFECT OF A SLIGHT DEVIATION FROM A HORIZONTAL FLIGHT PATH

When the term $[(L/D)_0 \sin \theta]$ is less than 0.100 Equations (5) and (5a) may be written in the form

$$\frac{V_H}{V_m} = 1.000 - K\left(\frac{L}{D}\right)_0 \sin \theta$$

or since θ is small

$$\frac{V_H}{V_m} = 1.000 - K\left(\frac{L}{D}\right)_0 \frac{h}{l} \tag{6}$$

Where h is the altitude lost or gained in the distance l, and K is a constant depending on the assumed variation of thrust with speed. For $T=0$ when $V=2V_m$ as in Equation (5) $K=0.333$.

For $T=T_0/2$ when $V=2V_m$ as in Equation (5a), $K=0.400$. h is negative if altitude is lost and positive if altitude is gained.

Figure 6 is a working diagram based on Equation (6) and the assumption that $T=0$ when $V=2V_m$. This diagram shows how the high speed is affected by a failure to maintain a true horizontal flight path during high-speed tests. Since the value of (L/D) at maximum speed is approximately 4 for the average airplane, a change of 40 feet in altitude per mile flown will change the maximum speed 1 per cent.

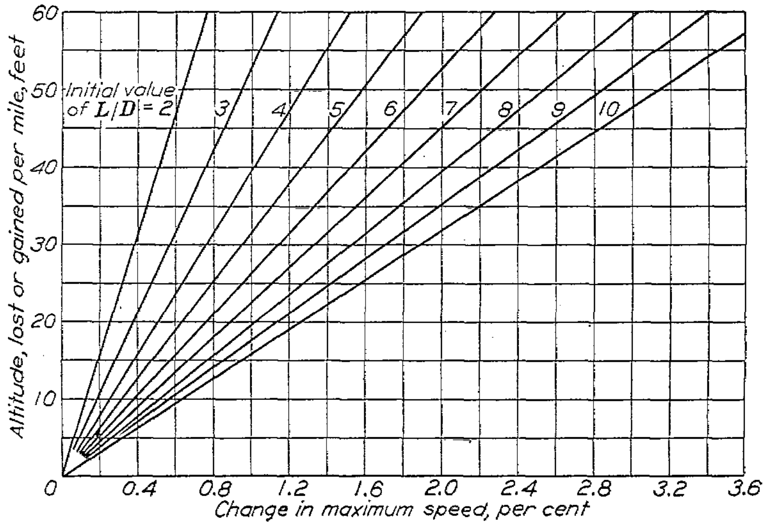

FIG. 6.—Effect of a slight inclination of the flight path on the maximum speed of airplanes.

The value of (L/D) at high speed may readily be determined for any airplane for which the maximum horizontal velocity V_m, brake horsepower (B.HP.), propeller efficiency η, and weight W are known. From the common power equation

$$D=\frac{375\ \eta\ \text{B.HP}}{V_m}$$

V_m being in M. P. H. and D in lb.
Since $W=L$

$$\frac{L}{D}=\frac{W}{D}=\frac{WV_m}{375\eta\ \text{B.HP}}=\frac{V_m}{375\eta}\left(\frac{W}{\text{B.HP}}\right) \tag{7}$$

Table VI contains calculations for the value of L/D at maximum speed for a number of typical airplanes. This value varies from 3.5 to 6.0. It has no direct relation to the maximum value of (L/D) or to the "fineness" of the design.

APPLICATIONS

This study was originally intended to show what increase in maximum horizontal velocity, as represented by the horizontal component of the flight velocity, could be obtained in an emergency, such as the landing of a slow airplane in a high wind. However, the data has been prepared and presented in such form as to make available information on a number of items of interest to pilots, as follows:

(1) Variation of flight velocity with inclination of flight path. See Tables I and III and Figures 1 and 3.

(2) Variation of effective horizontal velocity (i. e., horizontal component of flight velocity) with inclination of flight path. See Tables II and IV and Figures 2 and 4.

(3) Maximum effective horizontal velocity and angle of inclination of flight path which gives this maximum. See Figures 2, 4, and 5.
(4) Approximate value of the angle of inclination of the flight path at which horizontal component of the flight velocity becomes equal to the maximum velocity in horizontal flight. See Table V and Figures 2, 4, and 5.
(5) Effect of a small departure from a horizontal flight path (of particular interest to test pilots). See Figure 6.

CONCLUSIONS

It has been shown that for an average airplane having, for example, $L/D=4.5$ at high speed, the maximum value of the horizontal component of the flight velocity in a power glide, becomes 40 per cent greater than the maximum velocity in horizontal flight. This maximum value occurs when the flight path is about 30° to the horizontal. It has also been shown that the flight path must be inclined downward about 60° from the horizontal before the horizontal component of the flight velocity becomes less than the maximum velocity in horizontal flight.

The necessity for maintaining a strictly horizontal flight path in performance testing is clearly brought out by Figure 6, which shows that for the average airplane a loss of 40 feet in altitude per mile flown is sufficient to increase the maximum speed 1 per cent.

A few words of caution seem desirable at this time. Owing to the uncertainty of the thrust, the calculations made in this study should be considered as approximations only. There is ample reason to believe, however, that the effect of a power glide is substantially as indicated except at very high flight velocities.

TABLE I

VARIATION OF FLIGHT VELOCITY WITH INCLINATION OF FLIGHT PATH

$T=0$ when $V=2V_m$

$$\frac{V}{V_m}=\left[-\frac{1}{2}+\sqrt{2.25-\left(\frac{L}{D}\right)_0 \sin\theta}\right]$$

$\left(\frac{L}{D}\right)_0=$	2	3	4	5	6	7	8	9	10
Inclination of flight path θ	$\frac{V}{V_m}$								
+20	0.752	0.606	0.439	0.235	—	—	—	—	—
+15	.816	.714	.602	.478	0.335	0.162	—	—	—
+10	.880	.815	.747	.675	.599	.517	0.428	0.329	0.217
+5	.940	.910	.878	.847	.813	.780	.746	.710	.673
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
−5	1.057	1.085	1.112	1.139	1.165	1.191	1.217	1.242	1.267
−10	1.111	1.165	1.216	1.266	1.315	1.362	1.408	1.452	1.496
−15	1.164	1.240	1.313	1.383	1.451	1.517	1.579	1.640	1.700
−20	1.213	1.310	1.402	1.490	1.574	1.655	1.733	1.810	1.882
−25	1.259	1.375	1.485	1.590	1.688	1.783	1.873	1.960	2.045
−30	1.303	1.437	1.562	1.680	1.792	1.898	2.000	2.098	2.192
−35	1.343	1.493	1.632	1.763	1.886	2.003	2.115	2.223	2.327
−40	1.380	1.546	1.696	1.838	1.972	2.098	2.220	2.333	2.447
−50	1.445	1.632	1.806	1.966	2.116	2.260	2.396	2.524	2.648
−60	1.496	1.703	1.892	2.066	2.230	2.383	2.530	2.670	2.803
−70	1.532	1.752	1.951	2.135	2.310	2.472	2.626	2.773	2.913
−90	1.562	1.792	2.000	2.193	2.373	2.542	2.703	2.854	3.000

TABLE II

VARIATION IN HORIZONTAL COMPONENT OF FLIGHT VELOCITY WITH INCLINATION OF FLIGHT PATH

$T=0$ when $V=2V_m$

$$\frac{V_H}{V_m}=\left[-\frac{1}{2}+\sqrt{2.25-\left(\frac{L}{D}\right)_0\sin\theta}\right]\cos\theta$$

$\left(\frac{L}{D}\right)_0=$	2	3	4	5	6	7	8	9	10
Inclination of flight path θ	$\frac{V_H}{V_m}$								
+20	0.707	0.569	0.413	0.221	------	------	------	------	------
+15	.779	.690	.581	.462	0.324	0.157	------	------	------
+10	.867	.803	.736	.665	.590	.509	0.422	0.324	0.214
+5	.937	.907	.875	.844	.810	.777	.743	.707	.670
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
−5	1.053	1.081	1.108	1.135	1.161	1.187	1.213	1.237	1.263
−10	1.094	1.148	1.198	1.247	1.296	1.342	1.387	1.430	1.474
−15	1.125	1.198	1.268	1.337	1.402	1.466	1.526	1.584	1.642
−20	1.140	1.232	1.318	1.401	1.480	1.556	1.630	1.702	1.769
−25	1.142	1.247	1.346	1.442	1.530	1.616	1.688	1.777	1.855
−30	1.128	1.245	1.352	1.455	1.552	1.644	1.732	1.818	1.898
−35	1.101	1.223	1.337	1.445	1.545	1.642	1.733	1.822	1.907
−40	1.057	1.185	1.299	1.409	1.511	1.608	1.701	1.787	1.875
−50	.929	1.049	1.161	1.264	1.361	1.453	1.540	1.624	1.703
−60	.748	.852	.946	1.033	1.115	1.192	1.265	1.335	1.402
−70	.524	.599	.667	.730	.790	.845	.898	.948	.996
−90	.000	.000	.000	.000	.000	.000	.000	.000	.000

TABLE III

VARIATION OF FLIGHT VELOCITY WITH INCLINATION OF FLIGHT PATH

$T=\frac{T_0}{2}$ when $V=2V_m$

$$\frac{V}{V_m}=\left[-\frac{1}{4}+\sqrt{1.5625-\left(\frac{L}{D}\right)_0\sin\theta}\right]$$

$\left(\frac{L}{D}\right)_0=$	2	3	4	5	6	7	8	9	10
Inclination of flight path θ	$\frac{V}{V_m}$								
+20	0.687	0.482	0.191	------	------	------	------	------	------
+15	.772	.636	.476	0.268	0.042	------	------	------	------
+10	.852	.770	.682	.583	.472	0.389	0.167	------	------
+5	.928	.890	.851	.811	.769	.726	.680	0.632	0.581
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
−5	1.067	1.100	1.132	1.163	1.194	1.223	1.253	1.282	1.310
−10	1.131	1.194	1.252	1.309	1.363	1.416	1.467	1.517	1.565
−15	1.193	1.279	1.362	1.440	1.515	1.588	1.655	1.723	1.788
−20	1.248	1.358	1.461	1.558	1.652	1.738	1.822	1.904	1.982
−25	1.302	1.432	1.554	1.666	1.773	1.877	1.972	2.066	2.157
−30	1.351	1.500	1.637	1.767	1.887	2.000	2.108	2.213	2.312
−35	1.396	1.562	1.713	1.855	1.987	2.112	2.230	2.343	2.452
−40	1.438	1.618	1.783	1.936	2.078	2.213	2.340	2.461	2.577
−50	1.509	1.715	1.901	2.072	2.232	2.382	2.524	2.660	2.788
−60	1.565	1.796	1.993	2.177	2.350	2.512	2.664	2.810	2.950
−70	1.606	1.843	2.057	2.253	2.434	2.603	2.765	2.916	3.060
−90	1.638	1.887	2.110	2.323	2.500	2.667	2.842	3.000	3.150

TABLE IV

VARIATION IN HORIZONTAL COMPONENT OF FLIGHT VELOCITY WITH INCLINATION OF FLIGHT PATH

$T=\frac{T_0}{2}$ when $V=2\,V_m$

$$\frac{V_H}{V_m}=\left[-\frac{1}{4}+\sqrt{1.5625-\left(\frac{L}{D}\right)_0 \sin\theta}\right]\cos\theta$$

$\left(\frac{L}{D}\right)_0=$	2	3	4	5	6	7	8	9	10
Inclination of flight path θ	$\frac{V_H}{V_m}$								
+20	0.646	0.453	0.180	-------	-------	-------	-------	-------	-------
+15	.746	.614	.460	0.258	0.041	-------	-------	-------	-------
+10	.839	.758	.672	.574	.465	0.383	0.164	-------	-------
+5	.925	.886	.848	.808	.766	.723	.678	0.630	0.579
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
−5	1.063	1.096	1.128	1.158	1.190	1.218	1.248	1.277	1.305
−10	1.115	1.176	1.233	1.289	1.343	1.396	1.445	1.496	1.543
−15	1.152	1.235	1.315	1.392	1.463	1.535	1.599	1.665	1.728
−20	1.174	1.277	1.373	1.465	1.552	1.635	1.713	1.790	1.863
−25	1.180	1.298	1.408	1.510	1.606	1.700	1.786	1.872	1.954
−30	1.170	1.298	1.418	1.531	1.635	1.732	1.826	1.917	2.003
−35	1.143	1.280	1.403	1.518	1.628	1.729	1.826	1.918	2.008
−40	1.102	1.240	1.366	1.483	1.592	1.695	1.793	1.885	1.974
−50	.971	1.103	1.223	1.333	1.436	1.532	1.623	1.711	1.793
−60	.782	.898	.997	1.088	1.175	1.256	1.333	1.415	1.475
−70	.549	.630	.703	.771	.833	.891	.946	.997	1.046

TABLE V

MAXIMUM INCREASE IN HORIZONTAL VELOCITY AND CORRESPONDING INCLINATION OF FLIGHT PATH

$\left(\frac{L}{D}\right)_0$	Case I: $T=0$ when $V=2\,V_m$			Case II: $T=\frac{T_0}{2}$ when $V=2\,V_m$		
	Maximum value of $\frac{V_H}{V_m}$	Value of θ for maximum value of $\frac{V_H}{V_m}$	Approximate value of θ for $\frac{V_H}{V_m}=1.0$	Maximum value of $\frac{V_H}{V_m}$	Value of θ for maximum value of $\frac{V_H}{V_m}$	Approximate value of θ for $\frac{V_H}{V_m}=1.0$
2	1.143	23.5	44.8	1.180	24.8	48.3
3	1.248	26.5	53.0	1.302	27.6	55.6
4	1.353	28.5	57.7	1.418	29.5	59.9
5	1.455	30.2	61.1	1.532	30.5	63.1
6	1.553	31.5	63.6	1.636	31.3	65.4
7	1.646	32.5	65.6	1.737	32.0	67.3
8	1.736	33.3	67.3	1.832	32.4	68.7
9	1.823	33.9	68.7	1.925	32.7	69.9
10	1.907	34.5	69.8	2.012	33.0	70.7

V_H=horizontal component of flight velocity.
V_m=maximum velocity which can be sustained in horizontal flight.

TABLE VI

VALUE OF $\frac{L}{D}$ AT MAXIMUM SPEED

Airplane	Gross weight W lb.	Maximum speed V_m M. P. H.	B. HP.	Maximum propeller efficiency η_m	$\frac{L}{D}$ maximum speed
DH-4	4,300	124.0	420	0.75	4.5
VE-7	2,270	124.0	194	.77	5.0
SE-5	2,060	123.0	190	.77	4.6
MB-3	2,094	152.0	300	.80	3.5
MT	12,100	105.0	800	.76	5.6
JL-6 monoplane	3,600	111.0	240	.74	6.0
Sperry messenger	860	97.0	64	.78	4.5
Fokker D VII	2,460	151.0	350	.78	3.6
DT-2 seaplane	7,300	100.0	420	.71	6.5
TS-1 seaplane	2,125	122.0	208	.78	4.2
F5L boat	13,600	89.7	800	.68	6.0
R2C-1 land plane	2,300	248.0	500	.85	3.6

NOTE: $\frac{L}{D}$ at maximum speed must not be confused with the maximum value of $\frac{L}{D}$.